MAST-693 and CIEG-693 Final Exam May-21, 2018 19:00-21:00

A linear internal (semi-diurnal tidal) wave is generated with

$$\sigma_2 = 2\pi/T = 1.5\times10^{-4}\ s^{-1}\ \text{(wave period T=12 hours)}$$

off Oregon at 45° N latitude ($f=10^{-4}\ s^{-1}$). Vertical stratification is constant with $N^2=-(g/\varrho_0)\ \Delta\varrho/D = 2\times10^{-3}\ s^{-1}$. Both bottom and surface boundaries are present.

(1) What is the appropriate dispersion relation for such a wave? [10%]

(2) Are these dispersive or non-dispersive waves? [5%]

(3) Estimate mode-1 and mode-2 wave lengths $\lambda=2\pi/k$. [10%]

(4) Estimate the phase and group velocities of these waves (order of magnitude). [15%]

(5) Are these shallow or deep water waves as they propagate in water D=2000-m deep? [Recall shallow water waves have k*D<<1, where k is the wave number $k=2\pi/\lambda$.] [10%]

(6) Discuss how the wave reflects and refracts as it encounters the sloping bottom topography (assume constant slope) of the shelf break off Oregon. [20%]

(7) For a wave propagating from offshore towards the coast, what is the critical slope at which the onshore refracted wave disappears and all energy propagates offshore? [5%]

(8) Discuss, qualitatively or quantitatively, how wave properties change, if we consider a diurnal (T_1=24 hours and $\sigma_1 = \sigma_2/2 = 0.75\times10^{-4}\ s^{-1}$) as opposed to the semi-diurnal ($\sigma_2 = 1.5\times10^{-4}\ s^{-1}$) internal wave discussed above. [25%]

BONUS: What is the angle φ' that the wave number vector of the waves with σ_1 makes in an unbounded ocean against the vertical? Recall $\sigma=\sigma(f, N^2, \varphi')$. [10%]