MAST-693 and CIEG-693

Homework #1 due Feb.-24, 2020

1. (15 pts) Consider the two PDEs

$$\phi_t + \gamma\phi_{xxx} = 0$$

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Determine for each if (a) plane wave solutions (i.e., $\exp[i(kx - \omega(k)t])$ are permissible or if not why not, and (b) if so, what is the dispersion relationship $\omega = \omega(k)$.

2. (15 pts) Consider the 3-D second-order wave equation

$$\phi_{tt} - c^2\nabla^2\phi = 0$$

with plane wave solutions $\propto \exp[i(kx+ ly + mz- \omega t)]$. What is the dispersion relation $\omega = \omega(k, l, m)$.

3. (20 pts) For the 1-D 2nd-order wave equation on $-\infty < x < \infty$, consider the initial condition $\phi(x, t = 0) = \delta(x)$, where $\delta(x)$ is the Dirac delta function, and $\partial\phi(x, t = 0)/\partial t = 0$, what is the solution for $\phi(x, t)$? [Hint: $\delta(x)$ can be thought of as the Fourier transform of $\phi(x, t) = 1$]