Whitham (1974) Linear and Nonlinear Waves
Kinsman (1965) Wind Waves
Lighthill (1978) Waves in Fluids
Gill (1982) Atmosphere-Ocean Dynamics
Pedlosky (2003) Waves in Ocean and Atmosphere – Introduction to Wave Dynamics

1.0

Whitham: "...recognizable signal that is transfered from one part of a medium to another with recognizable velocity of propagation..." ← SIO quasi-soliton

- not just sinusoidal, but much more general
- not necessarily linear → bores + breaking dams + solitones (Conway / Severn Estuary)

- Space Scales
- Time Scales

NOT exact solutions of continuum formulation of mass + momentum eq. + thermodynamics
BUT approximate

↓

Linearized Field Equations → Fourier Superposition

10^{-9} sec	Radar	Remote Sensing Tool	Weather + Ice
10^{-3} sec	Acoustics	Communication, Sensing	
10^{+1} sec	Surface Gravity – Surf	no rotation	Surf
10^{+4} sec (½ d)	Surface Gravity – Tides	with rotation	Tides
10^{+6} sec (20d)	Vorticity Waves	with rotation	Currents

1.1 Plane Wave

$$\phi(\vec{x},t) = \mathrm{Re}\left\{ A\, e^{i(\vec{k}\cdot\vec{x} - \sigma t)} \right\}$$

$\phi(\vec{x},t)$ dependent variables velocity $\vec{u}$

pressure p

density ρ

A amplitude

$\vec{k} = (k, l, m)$ wave number

σ radian frequency

Other definitions

$\lambda = 2\pi / |\vec{k}|$ Wavelength

$f = \sigma / 2\pi$ Frequency

$T = 2\pi / \sigma = 1/f$ Period

Amplitude A is complex, hence it also contains phase information

$$\phi(\vec{x},t) = |A| \cos\left(\vec{k}\,\vec{x} - \sigma t + \tan^{-1}\left[\frac{\mathrm{Im}(A)}{\mathrm{Re}(A)} \right] \right)$$

Best to keep things complex and take real parts at very end

Examples:

$$\phi_t + c_0 \phi_x = 0$$

plane wave

$$\phi = \phi_0 e^{ikx - i\sigma t}$$

$$\phi_t = \phi_0 \cdot (-i\sigma) e^{ikx - i\sigma t} = (-i\sigma)\phi$$

$$\phi_x = \phi_0 (+ik) e^{ikx - i\sigma t} = (+ik)\phi$$

$\Downarrow \quad -i\sigma + c_0 ik = 0$

$\Downarrow \quad \boxed{\sigma = c_0 \cdot k}$ Dispersion Relation

phase speed $c_p \equiv \frac{\sigma}{k} = \frac{c_0 \cdot k}{k} = c_0$ constant for all k

group speed $c_g = \frac{\partial \sigma}{\partial k} = c_0$ constant for all k

<u>non</u>-dispersive waves

~~$+\phi\cdot\phi_x$~~ ✓

$$\phi_t + \phi_x - \phi_{xxt} = 0$$

linearized Benjamin-Bona-Mahony Eq.

$$\phi = \phi_0 \, e^{ikx - i\sigma t}$$

$$\phi_t = (-i\sigma)\,\phi$$
$$\phi_x = (+ik)\,\phi$$
$$\phi_{xxt} = (+ik)^2\,\phi\cdot(-i\sigma) = +i\sigma k^2$$

$$\boxed{\sigma = \frac{k}{1+k^2}}$$

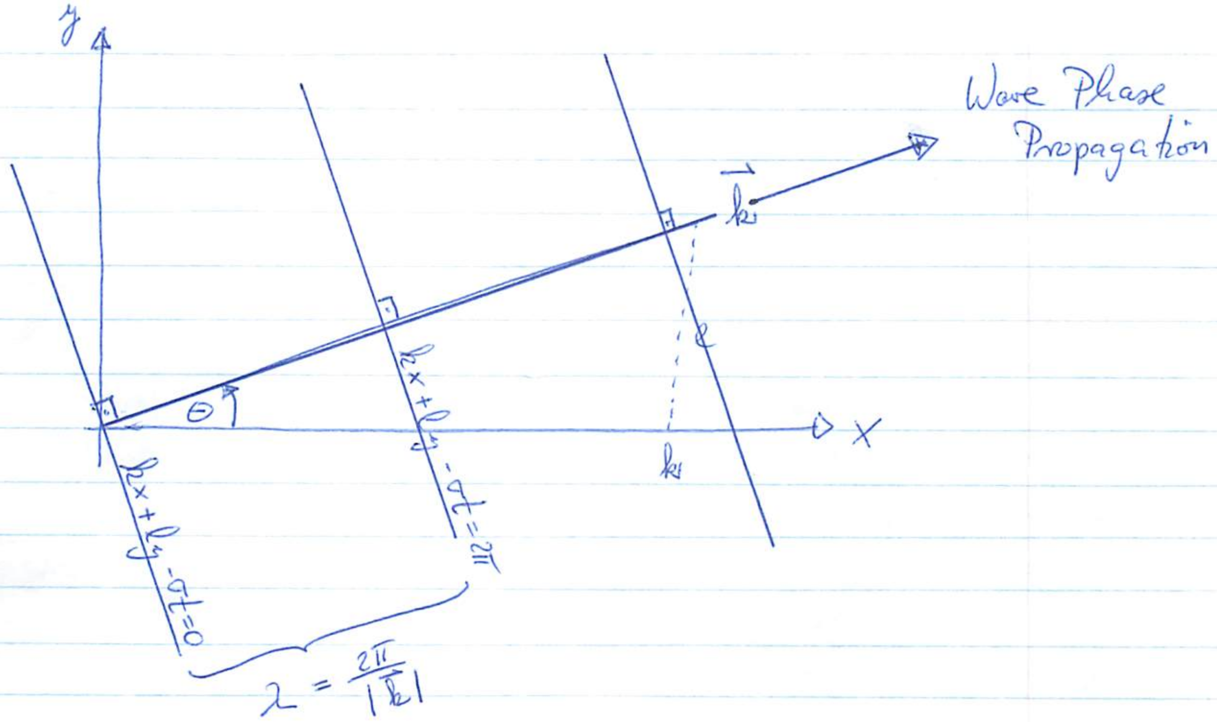
Dispersion Relation

phase speed $c_p = \dfrac{\sigma}{k} = \dfrac{1}{1+k^2} = c_p(k)$

group speed $c_g = \dfrac{\partial\sigma}{\partial k} = \dfrac{1\cdot(1+k^2) - k(2k)}{(1+k^2)^2} = \dfrac{1+k^2-2k^2}{(1+k^2)^2} = \dfrac{1-k^2}{(1+k^2)^2}$

$= c_g(k) \neq c_p(k)$

Traveling Plane Wave $A\, e^{i(\vec{k}\cdot\vec{x} - \sigma t)}$



$$c = \frac{\sigma}{|\vec{k}|} = \frac{\lambda}{T}$$ phase speed along $\vec{k}$

Note that phase speed in the x-direction is <u>NOT</u> $c \cdot \cos\theta$
BUT

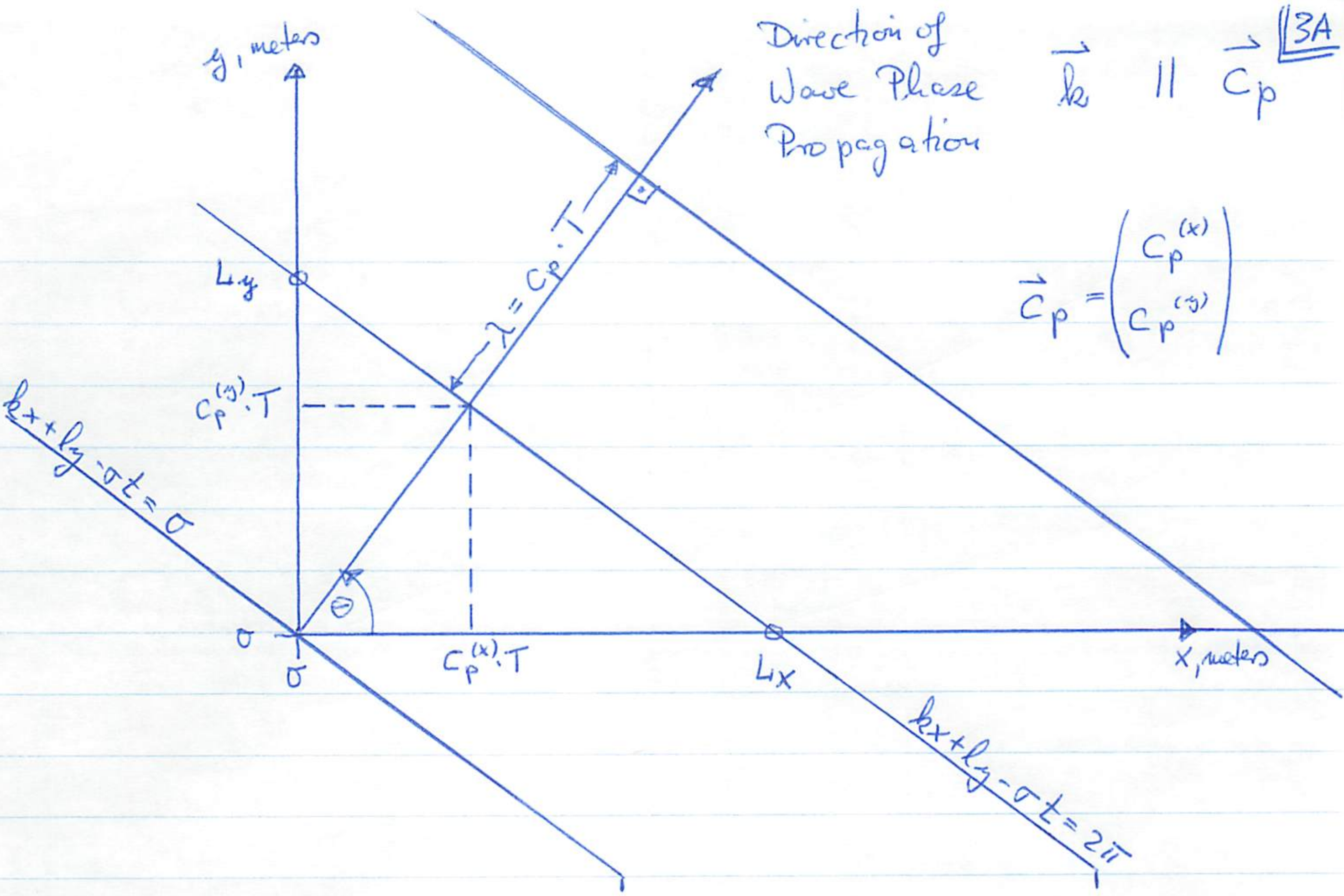
$$\frac{c}{\cos\theta} = \left(\frac{\sigma}{|\vec{k}|}\right) \Big/ \left(\frac{k}{|\vec{k}|}\right) = \frac{\sigma}{k}$$

which can be faster than c, for a wave traveling in y-direction

$\sigma / k \rightarrow \infty$

$\vec{k} \parallel \vec{C}_p$

$$\vec{C}_p = \begin{pmatrix} C_p^{(x)} \\ C_p^{(y)} \end{pmatrix}$$

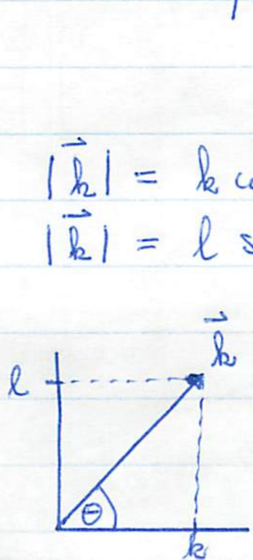


$L_x = \frac{\sigma}{k} \cdot T$ is the distance in the x-direction that the wave propagates in the x-direction in one period $T = 2\pi / \sigma$

$\lambda = |\vec{C}_p| \cdot T$ is the distance that the wave propagates its phase along its path in one wave period. This is the wavelength

$= 2\pi / |\vec{k}|$ is equal to 2π divided by the magnitude of the wavenumber vector, i.e., $|\vec{k}| = (k^2 + l^2)^{1/2}$

$$|\vec{k}| = k \cos\theta \qquad \text{or} \qquad \vec{k} = \begin{pmatrix} k \\ l \end{pmatrix} = |\vec{k}| \begin{pmatrix} 1/\cos\theta \\ 1/\sin\theta \end{pmatrix}$$
$$|\vec{k}| = l \sin\theta$$



$$\Downarrow \quad \vec{C}_p = \frac{\sigma}{|\vec{k}|} \cdot \frac{\vec{k}}{|\vec{k}|} = \frac{1}{|\vec{k}|} \begin{pmatrix} \sigma / \cos\theta \\ \sigma / \sin\theta \end{pmatrix}$$