6.1 (p. 149)

Topographic Rossby Waves

linear, hydrostatic, Boussinesq Approx.:

(1) $u_t - fv = -\frac{1}{\rho_0} p_x$

(2) $v_t + fu = -\frac{1}{\rho_0} p_y$

(3) $0 = -\frac{1}{\rho_0} p_z - g\rho/\rho_0$

(4) $\rho_t + w\rho_{0z} = 0$

$$\longrightarrow \boxed{w = -\frac{1}{\rho_0 N^2} p_{zt}}$$

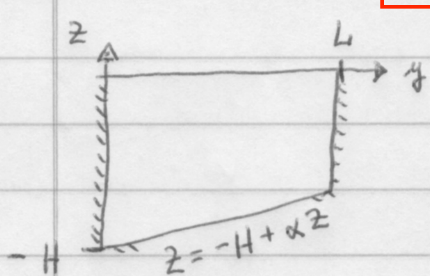
(5) $u_x + v_y + w_z = 0$

$\Longrightarrow$ **WAVE EQUATION**

$$\left[p_{xx} + p_{yy} + \left(\frac{\partial^2}{\partial t^2} + f^2\right)\left(\frac{p_z}{N^2}\right)_z \right]_t = 0$$

$e^{-i\sigma t}$:

$$\boxed{p_{xx} + p_{yy} + \left(f^2 - \sigma^2\right)\left(\frac{p_z}{N^2}\right)_z = 0}$$



bottom slope α

$v = 0$ @ $y = 0$ and $y = L$

$w = 0$ @ $z = 0$

$w = \alpha v$ @ $z = -H + \alpha z$

$y = 0, L \qquad v = 0 \qquad \Rightarrow \qquad \underline{i\sigma p_y + f p_x} = 0$

$z = 0 \qquad w = 0 \qquad \Rightarrow \qquad \underline{p_z = 0}$

$z = -H + \alpha z \qquad w = \alpha v \qquad \Rightarrow \qquad \underbrace{i\sigma(f^2 - \sigma^2) p_z}_{W} = \alpha N^2 \underbrace{(i\sigma p_y + f p_x)}_{V}$

Scale (x, y) by L
$\qquad\quad z$ by H

Set $\qquad \omega = \sigma / f$
$\qquad\quad N = \text{const.}$

Then our wave problem with BC becomes

$$p_{xx} + p_{yy} + \underbrace{\frac{(1-\omega^2)}{S'^2}} \underline{p_{zz}} = 0$$

$$i\omega p_y + p_x = 0 \qquad \text{at } y = 0, 1$$

$$p_z = 0 \qquad \text{at } z = 0$$

Bottom Boundary Condition: $\quad i\omega(1-\omega^2) p_z = \underline{\delta} S'^2 (i\omega p_y + p_x) \quad \text{at } z = -1 + \delta y$

where $\delta = \dfrac{\alpha L}{H}$ is scaled bottom slope

Internal Rossby radius

$\boxed{S' = (N \cdot H) / (f L)}$ is Burger number $S' = \boxed{\dfrac{N \cdot H}{f}} \Big/ L$

Burger number S' measures the importance of stratification relative to the spatial scale of the motion

$$S' = \frac{\text{internal Rossby radius}}{\text{geometric length scale}} = \frac{N \cdot H / f}{L}$$

Large S' means strong stratification $L_D > L$
(and/or large aspect ratio H/L)

Small S' means weak stratification $L_D < L$
(and/or nearly horizontal motions)

Consider $\omega \ll 1$ $(\sigma \ll f)$ low-frequency
$\delta \ll 1$ (gentle bottom slope)

$$p_{xx} + p_{yy} + \frac{1}{S'^2} p_{zz} = 0$$

$y = 0, 1$: $p_x = 0$

$z = 0$: $p_z = 0$

$z = -1$: $i \omega p_z = \delta S'^2 p_x$

Solution is

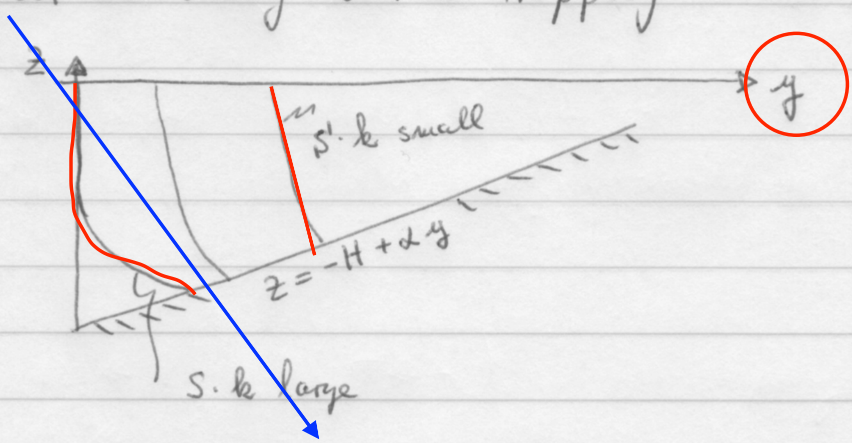
$$p = \underline{e^{ikx}} \; \underline{\sin(n\pi y)} \; \underline{\cosh(\mu z)}$$

where

$$\underline{\mu^2 = S'^2 (n^2\pi^2 + k^2)} \quad \text{gives a vertical decay scale}$$

Thus strong stratification and/or short spatial scales

result in strong bottom trapping



Dispersion relation becomes (non-dimensional)

$$\boxed{\omega = -\frac{\delta k S'^2}{\mu \tanh \mu}}$$

propagates with shallow water on the right (northern Hemisphere)

k is the across-slope component of the wave number vector

Compare this to the planetary Rossby Wave Dispersion

$$\cancel{\frac{\sigma}{f} = -\frac{\alpha L}{H} \frac{k^* \Delta S^2}{1} \frac{1}{S \mu^* \Delta \tanh \mu^* \Delta}}$$

For weak stratification $S' \to 0$

$s\,\mu \to 0$

$s\,\mu \tanh \mu \to \mu^2 = S'^2 (n^2\pi^2 + k^2)$

and

$$\omega = -\frac{\delta k \cancel{S'^2}}{\cancel{S'^2}(n^2\pi^2 + k^2)} = -\frac{\delta k}{n^2\pi^2 + k^2}$$

barotropic
topographic Rossby Waves

or dimensionally

$$\sigma = -\frac{\alpha}{H} k \cdot f \cdot \frac{1}{\left[(n\pi/L)^2 + k^2\right]}$$

k is the along-slope
wave number vector
component

This is the dispersion for topographic Rossby Waves

$$\boxed{\sigma = -\frac{\alpha k f}{H\left[(n\pi/L)^2 + k^2\right]}}$$

$$\sigma = -\frac{f\alpha k}{H} \cdot \frac{1}{(l^2 + k^2)}$$

$K^2 = l^2 + k^2 \gg \frac{f^2}{gH}$ horizontal wave ~~number~~ length much smaller than Rossby radius

$\frac{2\pi}{K} \ll \frac{\sqrt{gH}}{f}$ short waves