GFD I final exam, december 1987 Andreas Münchow

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Dear Matthew,

'hope the following notes and sketches are of some use for your attempt. The sketches should not be taken as exact representations of what actually happens but rather what might happen, i.e. the dominant flow in interior can flow against the wind direction. Actually such an ~~explanation~~ observation in the Pacific had some influence on the development of what is called Sverdrup dynamics (since) some 40 years ago.

Also within the Sverdrup frame it is not quite possible to decide where the mass and vorticity (eastern or Western Boundary) is balanced by completely different dynamics which are much more subtle (flow intensification). Hence take care, especially at the Western boundary where "Sverdrup" blows up. not really ~~Flat~~ (mathematically possible) intensification at the eastern boundary would blow up the Eastern and Western boundary Sverdrup regime.

Anyway also take care with time dependent processes, again ~~at~~ they are most prominently observed at Western Boundaries at ≈ 100 km off the coast. Rossby waves are dominant there and could cause trouble, also at your northern open boundary which - as a result - does not allow for leakage for the interior flow. At the eastern boundary ≈ 20-30 km slight boundary effects and Rossby and Kelvin waves could have an effect also.

OK, but only steady flow is at issue

Also, please excuse me that I am not quite on top, I am tired a little. Anyway, 'think there's a good class on more details and general ideas on the topic next semester here. Hope to see you then, yours Andreas

(1) Assume your flow in a flat, homogeneous ocean to be described by an angular momentum (≙ potential vorticity) balance between a steady blowing wind torque and the variation with latitude of the locally normal component of the rate of rotation:

$$v_0 = \vec{k} \cdot \text{curl}\, \vec{\tau} \quad ✓ \qquad \text{(Pedlosky, 1979, p. 224)}$$

v_0 is the first order velocity component in the North-South direction, it's a scaled unknown which together with pressure p and east-west velocity component u_0 has to be solved for. To lowest (~~first~~ zeroth) order* the flow is along lines of background or ambient vorticity. It is called - in the absence of any first order forcing - geostrophic flow. The dilemma with this flow is that it is degenerate~~d~~. We have only two equations for our three unknowns, hence we do not know why the flow is there or what is causing it. The additional statement about the conservation of potential vorticity (angular momentum), from which the above statement is an approximation, closes the dynamical system by considering ✓ forcing and higher order (smaller) effects.

* in Rossby number $\varepsilon \ll 1$,

In neglecting all other ~~effects~~ contributions to the potential vorticity balance ~~except the to~~ it was necessary that the "interior" or "depth averaged" or "geostrophic" velocity scale were

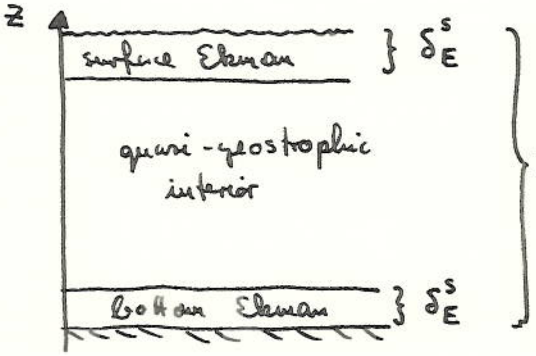
$$U = ✓\ \tau_0 / \rho \beta_0 D L = ✓\ 1 \text{ cm/sec}$$

for a windstress scale $\tau_0^{(1\,\text{dyne/cm}^2)}$, a waterdepth scale D (1 km), a lengthscale associated with the windstress ✓ L (1000 km), and the local normal rate of rotation f varying as

$$f = f_0 + \beta_0 y \qquad (\beta\text{-plane approximation})$$

with $f_0 \approx -10^{-4}\,\mathrm{sec}^{-1}$ and $\beta_0 \approx 10^{-13}\,\mathrm{cm}^{-1}\,\mathrm{sec}^{-1}$

The next step and helpful simplification ~~of~~ considering the physics involved is to consider the ocean to be divided into 3 different regimes or layers in the vertical z :



The (direct) action of the wind can be thought of as being restricted to a small Ekman layer of depth δ_E^s. Here turbulent frictional stresses are dominant and together with the coriolis force ~~form to~~ are a good approximation of the (linear) momentum balance. The same applies for the flow near the bottom where the geostrophic interior flow is slowed down by frictional forces to satisfy the boundary condition of no slip (no tangential velocity immediately at the bottom). Another assumption ($E_v^{1/2} = \delta_E / D \ll 1$) leads to the very useful result that the horizontal pressure gradients in the Ekman layers are those of the geostrophic (or should I say quasi geostrophic ?) interior.

A velocity scale for the ~~flow~~ ageostrophic flow in the Ekman layers can be found from a ~~momentum~~ momentum balance ~~between~~ due to frictional ~~stress variations~~ forces and coriolis ~~forces~~ forces:

$$\rho f u_E \sim \frac{\partial \tau}{\partial z} \cong \frac{\tau_0}{\delta_E} \quad \Rightarrow \quad u_E \sim \frac{2\tau_0}{\rho f \delta_E} = 10\ \mathrm{cm\,sec^{-1}}$$

A dimensional check would have shown missing ρ.

④

The stress τ_0 is taken as 1 dyne/cm², ~~[illegible]~~ the coriolis parameter $|f|$ was 10^{-4} sec and the depth is scaled with δ_E, the Ekman layer depth (20m)

The neglected terms in the vorticity balance all scaled as compared to v_0 (representing vortex tube stretching/squashing due to planetary vorticity changes ~~or~~ β-effect – and the wind stress curl) ~~as~~ with ✓

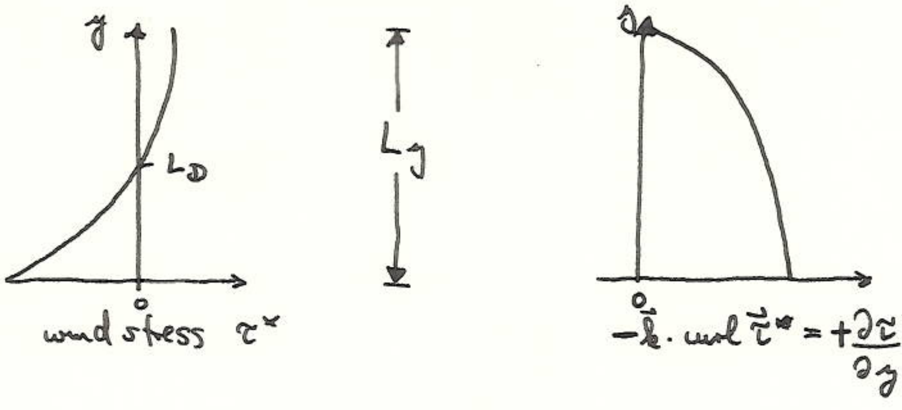
only interior vel. is relevant ✓

$$\beta^{-1} = \frac{U}{L^2 \beta_0} = \begin{cases} 10^{-3} & U = U_g = 1\,\text{cm sec}^{-1} \\ 10^{-2} & U = U_E = 10\,\text{cm sec}^{-1} \end{cases}$$

Therefore, ~~the~~ relative vorticity, vortex tube stretching or squashing due to free surface variation, and a frictional torque associated with the drag at the bottom all ~~can~~ are at least two order of magnitude smaller than the β effect and the wind torque at free surface.
The approximation made seems to be justified à posteriori.

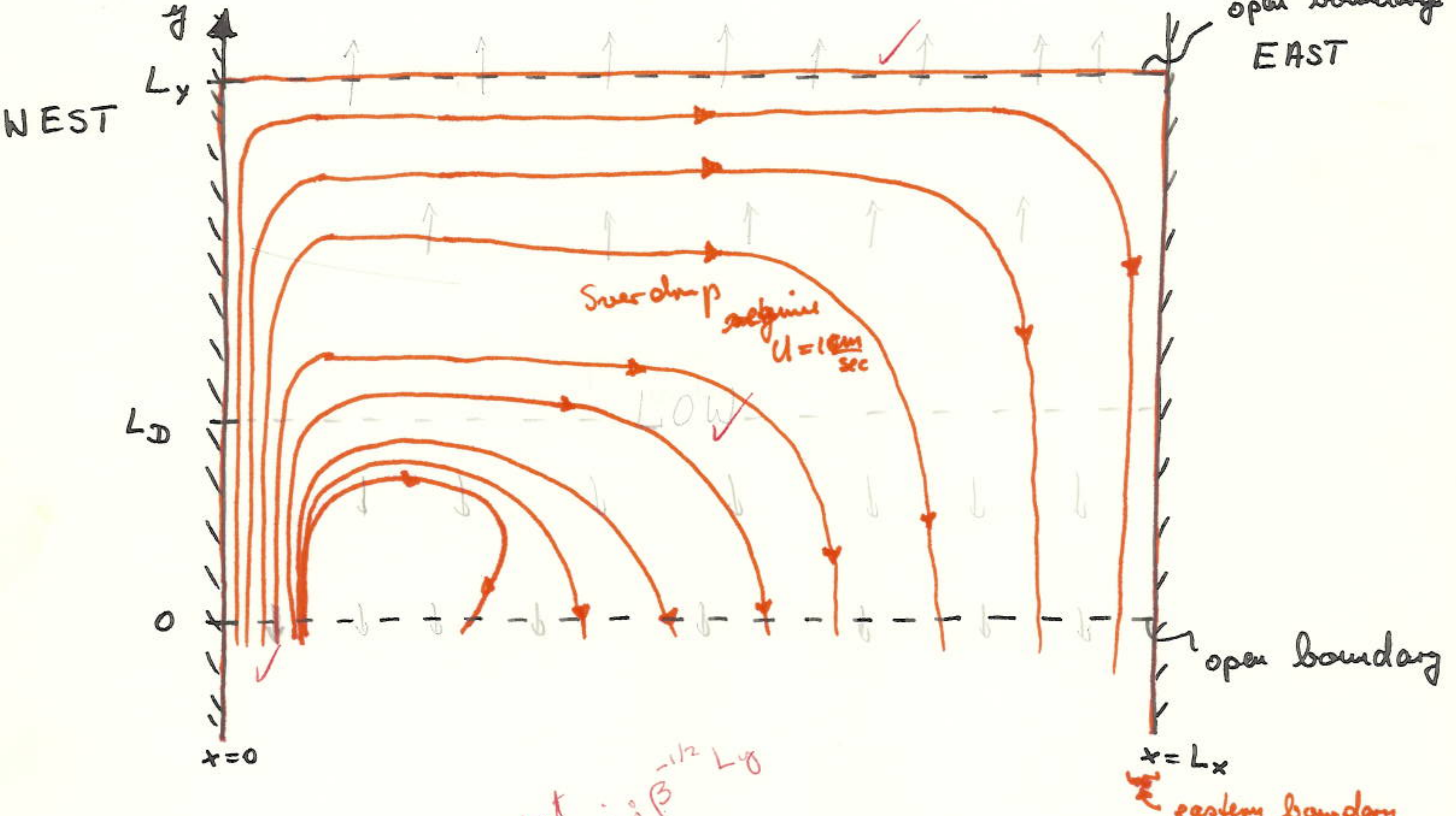
$$\frac{13}{15}$$

With a wind stress and windstress curl distribution as



Wind stress dir. is reversed from intended, but this won't create other than reversed flow direction.

I suggest the following large scale circulation pattern for the quasi-geostrophic interior in a basin in the southern hemisphere of the earth:



WBC, width δ_{BC} ~ say what scale this is: $\beta^{-1/2} L_y$
dependent upon
the "choosen"
dynamics here
(inertial B.C., that
is nonlinearity dominant, or lateral friction etc) No lat. fric. here.

streamlines in red (interior, quasi geostrophic)

surface Ekman mass flux in lead

— The circulation in the surface Ekman layer is driven by the wind. The resulting Ekman mass flux over the surface Ekman layer of thickness δ_E^s is to the left ('southern hemisphere'!) looking in the direction of the wind (formal $\vec{M}_E = \vec{\tau} \times \vec{k} / \rho f$, $f<0$). ✓

The windstress is positive for $y > L_D$ OK and negative for $y < L_D$, that is for $y > L_D$ the surface Ekman mass flux is to the North north of L_D and to the South south of L_D.

This results ~~to~~ in a divergent flow regime ✓ in the surface Ekman layer with Ekman ~~[illegible]~~ suction. Also, it is, for the same reason, a low pressure centre. ✓

The perturbation horizontal pressure gradient does not vary with depth in the Ekman layers because ~~they~~ ~~are~~ of the small Ekman number $E_V^{1/2} = \delta_E / D$. ✓

Geostrophic (interior) flow ~~also~~ is vertical uniform, too.

Therefore the low pressure over the model domain ✓ forces the interior to flow ~~to~~ clockwise. This orientation is the only one consistent with a low pressure system in the Southern hemisphere.

It is important to note that the Ekman suction of fluid from the quasi-geostrophic interior due to the divergent surface Ekman flux is the driving agent of the interior flow. ✓

— Next, the meridional velocity v_0 (north-south) is directly proportional to the wind stress curl, which in this case is the first derivative ~~of~~ $-\frac{\partial \tau}{\partial y}$. ✓

~~vor~~ This gradient is assumed to be zero at $y = L_y$, hence there is no quasi-geostrophic northward interior flow at that position. ~~It is a zon~~ The streamline is zonal here indicating a zonal flow only. If we would neglect boundary effects this would be a zero velocity stream line and there would be no ~~flow~~ interior flow at $y = L_y$. But boundary effects might render the situation, derivations from this ~~zero~~ interior flow at $y = L_y$ should be not surprising (Rossby waves transmitting energy to the east, short Rossby waves)

no. Only v_0 vanishes at L_y, not u_0

Your sketch is correct

— Because the windstess is a monotonically increasing function with latitude, ~~the proportional~~ the windstess curl is always negative over the entire modeled domain. Hence the meridional ~~zonal~~ flow in the interior v_0 is always to the south within the validity of Sverdrups balance. It is the windstress curl or torque which ~~for~~ drives a flow ~~cross iso ambient potential vorticity~~ across lines of ambient vorticity (the β-ramp here). The geostrophic flow would be entirely east-west, but the windstress curl provides the ~~torque~~. vorticity for a fluid particle to bend and, here, flow south and satisfy one boundary condition of no normal flow. It is not clear if this condition is to be satisfied at the Western or Eastern boundary. ~~It~~ It is clear, though.

no; u_0, v_0 are the geostr. vel. components.

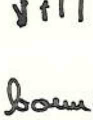
Important here is that vorticity input from the windstess curl is communicated to the interior via vertical suction or pumping (corresponding vortexe tube stretching or squashing respectively) of fluid. It is not the direct action of the wind driving the quasi-geostrophic interior flow, it is the "higher order dynamics" of the Ekman layer which transform the horizontal windstress ~~curl~~ into ~~a~~ a ~~F~~ vorticity source for the interior.

— The reason, that I have choosen ~~a~~ western boundary current to balance the mass fluxe over the model domain is the following:
Note first that the interior eastward circulation was a result of the low pressure centre due to the divergent surface Ekman fluxe. ~~Note also, dear Matthew, that as a result of the wa~~ due to the windstress. Note also, dear Matthew, that the Sverdrup balance supports with the given windstress only ✓ southward flow across iso-ambient-vorticity potentials due to the windstress curl.

no lat. friction, so no no-slip bds.

Hence only one ~~no slip and~~ no normal flow boundary condition can be satisfied within the steady Sverdrupian framework: the eastern boundary where the flow is to the south. Therefore I exepect the ~~solution~~ depitched solution to be valid very close to the eastern boundary.

OK

Very close to the boundaries, in any case (east or west), relative vorticity, velocity ? shear are getting important.
This offers another plausibility thought to place the ~~bread~~ dramatic breakdown of steady Sverdrupian flow ~~to~~ at the western instead of at the eastern boundary: consider a vorticity tendecy of a fluid particle east and west. The windstress curl is the same east and west, ~~A~~ no ✓ difference. To satisfy no slip boundary conditions the northward flow at the West ~~sor~~ induces a counter=clockwise rotation (or relative vorticity due to velocity ~~shear~~ shear : ). The situation is reversed at the

OK, but not possible here.

eastern boundary but leads to the same "vorticity tendecy": ). The only "vorticity" difference, preventing an easy vorticity balance to hold throughout the basin is the β-effect, because in the east particles move toward smaller ambient vorticity whereas in the West they move toward bigger ambient vorticity.

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- Also, our scaling and steady windstress lead us directly to a steady quasi-geostrophic flow and ~~or~~ a steady Ekman flow. If we are forced to allow for relative vorticity near the Western Boundary, then the flow is not necessarily time independent or linear anymore. Rossby waves might become important, especially at the western boundary, because they transmit energy or signal (i.e. frictional dissipation) only very slowly away from the boundary. ok By (3.14.25) only western b.l. is possible if flow is onshore. But yours is offshore, so generation of Rossby waves is indicated, as you suggest. You've made the best possible arguments, given the reversed $\vec{\tau}$.

(3) The Western Boundary current was introduced to mass and vorticity balanced in the interior, because all streamlines away from the Western Boundary, were leaving the domain. ✓

On the other side the quasi-geostrophic interior flow has to satisfy a no slip condition at the bottom, hence the interior is "creating" or is a driving agent for a bottom Ekman flow, which (due to the low pressure centre) results in a ~~flow~~ convergent Ekman bottom flow ✓ with ~~Ekm~~ fluid being pumped out ~~of the~~ into the interior. But both Ekman layer are connected with the surface and bottom layers outside the domain and also (see fig. 3) could possibly balance themselve, i.e. the interior gain of fluid from the bottom is lost to the surface ok Ekman layer. Also the ~~dre~~ direct contribution in terms of mass fluxes from the Ekman layers should be negligible

⑩

if compared with the geostrophic first order mass flux out of the model domain. actually the surface E.L. vert. flux will make a difference, but the central point is that the WBL balances most of the mass flow.

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(4) The wind is normal to both coasts, hence I do not expect considerable up or downwelling ✓ at the coast. Nevertheless in both bottom and surface Ekman layers the velocity spirals (Ekman spiral), that is within the Ekman layers locally there ~~[illegible]~~ are velocity components at some depth which violate the no normal flow at boundaries. These are the only possibilities I can think about which could result to (local) up or downwelling. This "welling" is balanced entirely in the Ekman layers. ✓

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(5) A meridional cut through the model domain at $x = L_x/2$ ~~repres~~ is ~~depa~~ derived from the formerly described dynamics:

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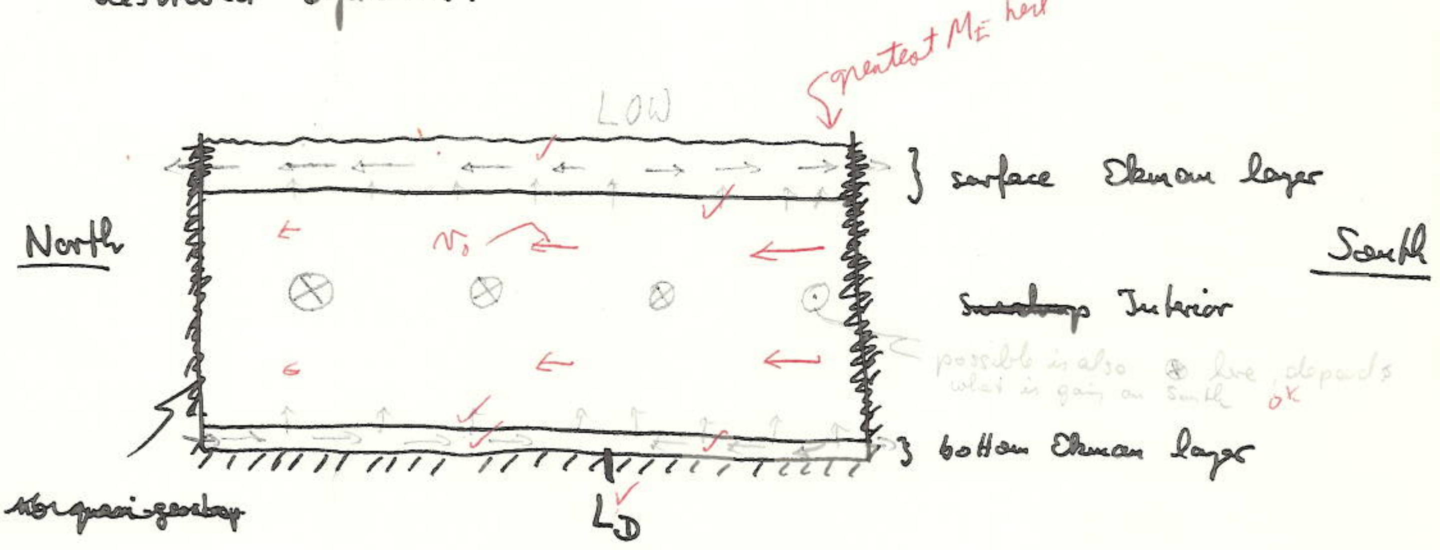

The flow is not symmetric in East - West ✓ direction nor ~~is it~~ it is ~~North~~ North - South.
The interior relative ($\overset{\text{zeroth}}{\text{first}}$ order) vorticity is clockwise, that is negative. ✓