Homework-05 Due Mon Oct.-31, 2005 (prior to class)

1. The steady state Ekman equations on an f-plane of constant depth H

$$-fv = Au_{zz}$$
$$fu = Av_{zz}$$

subject to boundary conditions

$$u=v=0 \text{ at } z=0$$
$$u=U \ v=0 \text{ at } z=+\infty$$

describe the detailed spatial structure of a bottom boundary layer forced by a spatially uniform (geostrophic) flow U above the boundary layer.

(a) Find a single fourth-order differential equation by placing the expression of u from the y-momentum into the x-momentum equation. [5pts]
(b) Find the solutions for (u,v) that satisfies the boundary conditions. [5pts]
(c) Calculate the relative vorticity vector for this problem (horizontal and vertical components) and discuss the statement that the vertical derivative of the momentum equations in terms of horizontal relative vorticity components describes "…a balance between the vorticity diffusion from the boundary and the compensating tilting of the planetary vorticity filaments…" [5pts]
(d) Find a spin-down time-scale T_e=W'/K for the uniform geostrophic flow U of height H by calculating the ratio between the rate of work W' done by the pressure force in the Ekman layer (Ekman velocity times horizontal pressure gradient integrated from the bottom to infinity) and the kinetic energy K of that geostrophic flow of uniform height H. [5pts]

2. (from Cushman-Roisin, 1994) Between 15N and 45N latitude, the winds over the North pacific consist mostly of easterly trades (15N-30N) and westerlies (30N-45N). An adequate representation is

$$\tau^{(x)} = \tau_0 \sin(\pi y/2L),\ \tau^{(y)}=0,\ -L<y<L$$

where to=0.15N/m2 is the maximum wind stress and L=1670 km. Taking r_0=1028 kg/m^3 and the value of the Coriolis parameter at 30N, calculate the vertical Ekman pumping velocity. Which way is it directed? Calculate the vertical volume flux over the entire 15N-45N latitude band of the North Pacific (with=8700 km). Express your answer in units of sverdrup (1 sverdrup = 10^6 m^3/s). [10pts]

Gill (1982), chapters 9.2, 9.3, and 9.4: excellent discussion of the essentials of Ekman dynamics and Ekman pumping;
Pedlosky (1979), chapter 4.3: excellent and exceptionally clear discussion of Ekman layers.