Homework-07 Due Tue Nov.-22, 2005

1. Show that the solutions to the (nonlinear) geostrophic adjustment problem

$$
\begin{aligned}
& \left.h(x)=H\left[1-\exp \left(x-L_{D}\right) / L_{\mathrm{D}}\right)\right] \\
& \left.\mathbf{v}(\mathbf{x})=-\left(\mathrm{g}^{\prime} H\right)^{1 / 2} \exp \left(x-L_{\mathrm{D}}\right) / \mathbf{L}_{\mathrm{D}}\right)
\end{aligned}
$$

are consistent with the thermal wind equation

$$
\partial_{z} v=-g /\left(\rho_{0} f\right) \partial_{x} \rho
$$

which writes in discrete form (Margules relation)

$$
\Delta v / \Delta z=-g /\left(\rho_{0} f\right) \Delta \rho / \Delta x
$$

where the variable $h$ is the thickness of the upper layer of density $\rho$ overlaying a lower layer of density $\rho_{0}$, v is the along-front velocity, $\mathrm{L}_{\mathrm{D}}=\left(\mathrm{g}^{\prime} \mathrm{H}\right)^{1 / 2} / \mathrm{f}$ is the internal Rossby radius of deformation, and $g^{\prime}=g \Delta \rho / \rho_{0}$ is the reduced gravity [10 pts].
2. Please find and correct ALL errors in the statement:
"...the thermal wind relation $\partial_{z} \mathrm{u}=\mathrm{g} /\left(\rho_{0} \mathrm{f}\right) \partial_{\mathrm{x}} \rho$ gives the horizontal velocity shear as a function of depth. A reference velocity is not necessary to estimate this quantity, however, one must prescribe a vanishing pressure gradient at the bottom to estimate a geostrophic velocity in a contineously stratified fluid..." [5pts]
3. Below you find a graphical presentation of the density anomaly $\rho-1000 \mathrm{~kg} / \mathrm{m}^{3}$ (contours) and temperature (colors) fields taken during a survey Sept. 22-26, 2004 near 73N latitude in the Arctic Ocean (courtesy of Dr. Pickart, WHOI, see http://www.whoi.edu/arcticedge/arctic west04/update/ for a narrative). Use the thermal wind relation to roughly estimate the geostrophic velocity field associated with this feature assuming that the horizontal pressure gradient vanishes at $300-\mathrm{m}$ depth. More specifically, estimate currents relative to no flow at $300-\mathrm{m}$ at distances $(5,15,25) \mathrm{km}$ and depths $(50,100,150,250) \mathrm{m}$ and sketch the results. What is the Rossby number (relative to planetary vorticity) of this feature. [10 pts]


